Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Tuesday 18 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Answer all questions. Write your answers in the spaces provided.

1. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$\boxed{a_{n+1} = 4 - a_n}$$

$$a_1 = 3$$

Starting point

recursive sequence (plugging in a term on the right gives the next term on the left)

Find the value of

(a) (i) a_2 → $a_2 = a_{1+1}$ $(n=1)$

(ii) a_{107}

end

(2)

(b) $\sum_{n=1}^{200}(2a_n - 1)$ → terms

starrirt value

(2)

(a) This is a recursive sequence:

$a_{n+1} = 4 - a_n$ → notice how $n+1$ is 1 more than n

Let's find:

(i) a_2:

$a_2 = 4 - a_1 = 4 - 3 = 1$

(ii) a_{107}:

We cannot find a_{107} directly using the given formula. BUT:

$a_3 = 4 - a_2 = 4 - 1 = 3$

$a_4 = 4 - a_3 = 4 - 3 = 1$

$\underset{a_1}{3}, \underset{a_2}{1}, \underset{a_3}{3}, \underset{a_4}{1} \ldots$

The series repeats itself every $\boxed{2}$ terms; it is periodic.

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Question 1 continued

$\underbrace{3, 1}_{1}, \underbrace{3, 1}_{2}, \underbrace{3, 1}_{3} \ldots$

so : $\dfrac{107}{2 \text{ terms}} = 53 + 1$

remainder, one more term

53 repeats of 3,1

$\underbrace{3, 1}_{1} \; \underbrace{3, 1}_{2} \; \underbrace{3, 1}_{3}, \ldots, \underbrace{3, 1}_{53}, 3$

one more!

so : $a_{107} = 3$

(b)

$\sum_{n=1}^{200} (2a_n - 1)$ means start the sequence at $n=1$ and sum all the terms up to $n = 200$

$= (2a_1 - 1) + (2a_2 - 1) + (2a_3 - 1) + \ldots + (2a_{200} - 1)$

Since the series repeats itself :

$2(3) - 1 = 5$

$2(1) - 1 = 1$

$\underbrace{5 + 1} + \underbrace{5 + 1} + \underbrace{5 + 1} + \ldots$

we end at $n = 200$: $\dfrac{200}{2 \text{ terms}} = 100$ repeats $(5+1)$

$\underbrace{5 + 1}_{1} + \underbrace{5 + 1}_{2} + \ldots + \underbrace{5 + 1}_{100}$

So sum up to $a_{200} = 100(5+1)$

$= 600$

Q1

(Total 4 marks)

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2. A circle C has equation

$$x^2 + y^2 + 4x - 10y - 21 = 0$$

Find

(a) (i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The point $P(5, 4)$ lies on C.

(b) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

(a) Standard form of equation of a circle:

$$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$$

↳ rewrite given eq. into standard form:

$$x^2 + 4x + y^2 - 10y - 21 = 0$$

(1) complete the square for terms with x:

$$x^2 + 4x + y^2 - 10y = 21$$

↳ add $\left(\frac{b}{2}\right)^2$ to both sides:

$$x + 4x + \left(\frac{4}{2}\right)^2 + y^2 - 10y = 21 + \left(\frac{4}{2}\right)^2$$

$$x + 4x + 4 + y^2 - 10y = 21 + 4$$

$$x + 4x + 4 + y^2 - 10y = 25$$

↳ factorise

$$(x+2)^2 + y^2 - 10y = 25$$

Leave blank

Question 2 continued

(3) complete the square for terms with y:

$$(x+2)^2 + y^2 - 10y = 25$$

↳ add $\left(\frac{b}{2}\right)^2$ to both sides:

$$(x+2)^2 + y^2 - 10y + \left(\frac{-10}{2}\right)^2 = 25 + \left(\frac{-10}{2}\right)^2$$

$$(x+2)^2 + y^2 - 10y + 25 = 25 + 25$$

↳ factorise

$$(x+2)^2 + (y-5)^2 = 50$$

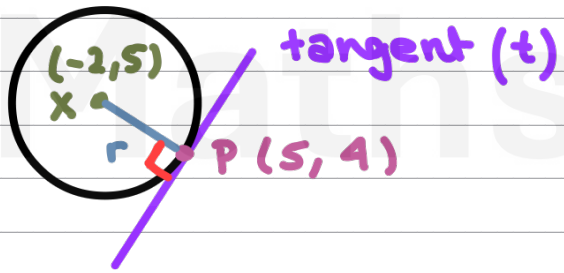
so: $(x \overset{-(-)}{+} 2)^2 + (y-5)^2 = 50$

$$(x-(-2))^2 + (y-5)^2 = 50$$

(i) coords of centre $= (-2, 5)$

(ii) radius $= \sqrt{50} = 12$

(b)



eq. of tangent → $y = mx + c$

radius XP is perpendicular to tangent at P

so: $m_{tangent} = -\frac{1}{m_r}$

Q2

(Total 7 marks)

P 6 2 7 2 1 A 0 5 3 2

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Question 2 continued

$$m_r = \frac{y_2 - y_1}{x_2 - x_1}$$

use : $(5,4)$ and $(-2,5)$

$$m_r = \frac{5-4}{-2-5} = -\frac{1}{7}$$

$$m_{tangent} = \frac{-1}{-1/7} = 7$$

$$y = mx + c$$

$$y = 7x + c$$

FINDING c :

Method 1 : PLUG $(4,-1)$ IN $y = mx + c$, FIND c

$$y = 7x + c$$
$$4 = 7(5) + c$$
$$4 = 35 + c$$
$$c = -34$$

So : $\boxed{y = 7x - 34}$ → eq. of tangent

OR :

★ Method 2 : USE : $y - y_1 = m(x - x_1)$

$$y - 4 = 7(x - 5)$$
$$y - 4 = 7x - 35$$
$$\boxed{y = 7x - 31}$$

note : this method is faster, gives equation directly !

Leave blank

3. (i) Use algebra to prove that for all real values of x

$$(x-4)^2 \geqslant 2x-9$$

(3)

(ii) Show that the following statement is untrue.

$2^n + 1$ is a prime number for all values of n, $n \in \mathbb{N}$

(1)

(i) $(x-4)^2 \geqslant 2x-9$

• expand $(x-4)^2$:

$x^2 - 8x + 16 \geqslant 2x - 9$

• move all terms to one side :

$x^2 - 8x + 16 \geqslant 2x - 9$

$x^2 - 8x - 2x + 9 + 16 \geqslant 0$

$x^2 - 10x + 25 \geqslant 0$

• factorise the inequality :

$x^2 - 10x + 25 \geqslant 0$

$(x-5)(x-5) \geqslant 0$

$(x-5)^2 \geqslant 0$

we know that square numbers are always greater than or equal to zero

so : $(x-5)^2 \geqslant 0$ is true

hence : $(x-4)^2 \geqslant 2x-9$ is true

(ii) Here, you can give a counter-example :

ex : $n = 3 \longrightarrow 2^3 + 1 = 9$

$= 3 \times 3$ so NOT PRIME

$n = 5 \longrightarrow 2^5 + 1 = 33$

$= 3 \times 11$ so NOT PRIME

Leave blank

4. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{4}x\right)^6$$

(4)

(b) Given that x is small, so terms in x^4 and higher powers of x may be ignored, show

$$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = a + bx^2$$

where a and b are constants to be found.

(3)

(a) Binomial expansion formula :

$$\boxed{(a+bx)^n = {}^nC_0 a^n (bx)^0 + {}^nC_1 a^{n-1}(bx)^1 + {}^nC_2 a^{n-2}(bx)^2 + \dots}$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n : power of binomial

r : (position of term) − 1

OR : USE CHOOSE FUNCTION ON CALCULATOR

for $\left(2 - \frac{1}{4}x\right)^6$:

$$= {}^6C_0 2^6 \left(\frac{-x}{4}\right)^0 + {}^6C_1 2^{6-1}\left(\frac{-x}{4}\right)^1 + {}^6C_2 2^{6-2}\left(\frac{-x}{4}\right)^2 + {}^6C_3 2^{6-3}\left(\frac{-x}{4}\right)^3$$

${}^6C_0 = 1$

${}^6C_1 = 6$

${}^6C_2 = \frac{6!}{(6-2)!\,2!} = 15$

${}^6C_3 = \frac{6!}{(6-3)!\,3!} = 20$

NOTES :-

- the first and last ${}^nC_r$ are always = 1
- ${}^nC_1$ is always equal to n
- BINOMIAL COEFFICIENTS ARE SYMMETRIC for $(a+x)^6$:
  ${}^6C_0 = {}^6C_6$ ${}^6C_2 = {}^6C_4$
  ${}^6C_1 = {}^6C_5$

P 6 2 7 2 1 A 0 8 3 2

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Question 4 continued

$$= (1)(64) + (6)(32)\left(\frac{-x}{4}\right) + (15)(16)\left(\frac{x^2}{16}\right) + (20)(8)\left(\frac{-x^3}{64}\right)$$

$$= 64 - 48x + 15x^2 - \frac{5}{2}x^3$$

(b) look at the two fractions :

$$\left(2 - \frac{1}{4}x\right)^6$$

$$\left(2 + \frac{1}{4}x\right)^6$$

• only the sign between terms changed from −ve to +ve.

Therefore, the expansion of $(2 + \frac{1}{4}x)^6$ is the same as the expansion of $(2 - \frac{1}{4}x)^6$, except all terms become positive :

$$\left(2 - \frac{1}{4}x\right)^6 = 64 - 48x + 15x^2 - \frac{5}{2}x^3$$

$$\left(2 + \frac{1}{4}x\right)^6 = 64 + 48x + 15x^2 + \frac{5}{2}x^3$$

$$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 \rightarrow \text{add the expansions!}$$

cancel

$$64 - 48x + 15x^2 - \frac{5}{2}x^3 + 64 + 48x + 15x^2 + \frac{5}{2}x^3$$

cancel

$$= 128 + 30x^3$$

Q4

(Total 7 marks)

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5. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and £x is the selling price of the watch.

According to this model,

(a) find, using calculus, the maximum possible annual profit.

(6)

(b) Justify, also using calculus, that the profit you have found is a maximum.

(2)

(a) maximum profit = maximum point, which occurs when:

$$\frac{dP}{dx} = 0$$

DIFFERENTIATION:

$$\boxed{y = ax^n \qquad \frac{dy}{dx} = nax^{n-1}}$$

$$P = 12x - x^{3/2} - 120$$

$$\frac{dP}{dx} = (1)(12)x^{1-1} - \left(\frac{3}{2}\right)x^{3/2-1} - (1)120x^{1-1}$$

$$\frac{dP}{dx} = 12 - \frac{3}{2}x^{1/2}$$

$$12 - \frac{3}{2}x^{1/2} = 0$$

$$\frac{2}{3} \times (12) = \left(\frac{3}{2}x^{1/2}\right) \times \frac{2}{3}$$

$$8 = x^{1/2}$$

Leave blank

Question 5 continued

$(8)^2 = (x^{1/2})^2 \rightarrow (x^a)^b = x^{ab}$

$x =$ £ 64 per watch

maximum selling price is P at $x = 64$

$P = 12(64) - 64^{3/2} - 120$

$P = 768 - 512 - 120$

$= £\ 136\ 000$

(b) if $\frac{d^2P}{dx^2} > 0 \rightarrow$ minimum point

$\frac{d^2P}{dx^2} = 0 \rightarrow$ point of inflection

$\frac{d^2P}{dx^2} < 0 \rightarrow$ maximum point

$\frac{dP}{dx} = 12 - \frac{3}{2}x^{1/2}$

$\frac{d^2P}{dx^2} = \frac{1}{2}\left(\frac{3}{2}\right)x^{1/2-1}$

$\frac{d^2P}{dx^2} = -\frac{3}{4}x^{-1/2}$

At $x = 64$: $\frac{d^2P}{dx^2} = -\frac{3}{4}(64)^{-1/2} = -0.09375$

$-0.09375 < 0$; it is a maximum

Q5

(Total 8 marks)

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6. $f(x) = kx^3 - 15x^2 - 32x - 12$ where k is a constant

Given $(x - 3)$ is a factor of $f(x)$,

(a) show that $k = 9$ **(2)**

(b) Using algebra and showing each step of your working, fully factorise $f(x)$. **(4)**

in degrees!

(c) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$$

giving your answers to one decimal place. **(2)**

(a) FACTOR THEOREM :

If $(x-p)$ is a factor of $f(x)$, $f(p) = 0$.

so: $(x-3)$ is a factor ; $f(3) = 0$

• SOLVE $f(3) = 0$:

$f(x) = kx^3 - 15x^2 - 32x - 12$

$f(3) = k(3)^3 - 15(3)^2 - 32(3) - 12 = 0$

$27k - 135 - 96 - 12 = 0$

$27k - 243 = 0$

$27k = 243$

$k = 9$

(b) Use algebraic division to factorise :

$$x - 3 \overline{\big) 9x^3 - 15x^2 - 32x + 12}$$

(1) Divide first term of dividend by first term of divisor : $\frac{9x^3}{x} = 9x^2 \rightarrow$ first term of quotient

$9x^2$

$$x - 3 \overline{\big) 3x^3 - 8x^2 - 20x - 12}$$

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Question 6 continued

(2) Multiply out and write under:

$$
\begin{array}{r|l}
 & 9x^2 \\
\hline
x-3 & 9x^3 - 15x^2 - 32x - 12 \\
 & 9x^3 - 3x^2 \\
\hline
\end{array}
$$

(3) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & 9x^2 \\
\hline
x-3 & 9x^3 - 15x^2 - 32x - 12 \\
 & -(9x^3 - 27x^2) \\
\hline
 & 12x^2 - 32x
\end{array}
$$

(4) Divide first term of expression by first term of divisor: $\frac{12x^2}{x} = +12x \rightarrow$ next term of quotient

$$
\begin{array}{r|l}
 & 9x^2 + 12x \\
\hline
x-3 & 9x^3 - 15x^2 - 32x - 12 \\
 & -(9x^3 - 27x^2) \\
\hline
 & 12x^2 - 32x
\end{array}
$$

(5) Multiply out and write under:

$$
\begin{array}{r|l}
 & 9x^2 + 12x \\
\hline
x-3 & 9x^3 - 15x^2 - 32x - 12 \\
 & -(9x^3 - 27x^2) \\
\hline
 & 12x^2 - 32x \\
 & 12x^2 - 36x \\
\hline
\end{array}
$$

(6) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & 9x^2 + 12x \\
\hline
x-3 & 9x^3 - 15x^2 - 32x - 12 \\
 & -(9x^3 - 27x^2) \\
\hline
 & 12x^2 - 32x \\
 & -(12x^2 - 36x) \\
\hline
 & 4x - 12
\end{array}
$$

Leave blank

Question 6 continued

(7) Divide first term of expression by first term of divisor : $\frac{4x}{x} = 4$ → next term of quotient

$$
\begin{array}{r|l}
 & 9x^2 + 12x + 4 \\
\hline
x - 3 & 9x^3 - 15x^2 - 32x - 12 \\
 & -(9x^3 - 27x^2) \\
\hline
 & 12x^2 - 32x \\
 & -(12x^2 - 36x) \\
\hline
 & 4x - 12
\end{array}
$$

(8) Multiply out and write under :

$$
\begin{array}{r|l}
 & 9x^2 + 12x + 4 \\
\hline
x - 3 & 9x^3 - 15x^2 - 32x - 12 \\
 & -(9x^3 - 27x^2) \\
\hline
 & 12x^2 - 32x \\
 & -(12x^2 - 36x) \\
\hline
 & 4x - 12 \\
 & 4x - 12
\end{array}
$$

(9) Subtract :

$$
\begin{array}{r|l}
 & 9x^2 + 12x + 4 \\
\hline
x - 3 & 9x^3 - 15x^2 - 32x - 12 \\
 & -(9x^3 - 27x^2) \\
\hline
 & 12x^2 - 32x \\
 & -(12x^2 - 36x) \\
\hline
 & 4x - 12 \\
 & 4x - 12 \\
\hline
 & 0
\end{array}
$$

So: $f(x) = (x-3)(9x^2 + 12x + 4)$ → factorise further !

$= (x-3)(3x+2)^2$

P 6 2 7 2 1 A 0 1 4 3 2

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Question 6 continued

(c) $9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$

$9x^3 - 15x^2 - 32x - 12 = 0$

so $x = \cos\theta$

therefore, we can rewrite:

$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$

$(\cos\theta - 3)(3\cos\theta + 2)^2 = 0$

$\cos\theta - 3 = 0$ REJECT!

$\cos\theta = 3 \rightarrow$ range of $y = \cos\theta$ is $-1 < \theta < 1$

OR

$3\cos\theta + 2 = 0$

$\cos\theta = -\frac{2}{3}$

SOLVE: $\cos\theta = -2/3$, $0 \leq \theta < 360°$

ON CALCULATOR:

$\theta = \cos^{-1}\left(-\frac{2}{3}\right) = 131.8°$ (1 d.p)

ON UNIT CIRCLE:

where $\cos\theta = -2/3$,

$\theta = 131.8°$

$\theta = 360 - 131.8°$

$= 228.8°$

So:

$\theta = 131.8°, 228.8°$

FINAL ANSWER!

(Total 8 marks)

Q6

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7. Kim starts working for a company.

- In year 1 her annual salary will be £16 200 a_1
- In year 10 her annual salary is predicted to be £31 500 a_{10}

Model A assumes that her annual salary will increase by the same amount each year.

arithmetic series

(a) According to model A, determine Kim's annual salary in year 2.

geometric series **(3)**

Model B assumes that her annual salary will increase by the same percentage each year.

(b) According to model B, determine Kim's annual salary in year 2. Give your answer to the nearest £10

partial sum **(3)**

(c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10

(3)

(a) In model A, we can find d, then use it to find salary in year 2:

year : 1 , 2 , ... , 10

salary : £16200 , a_2 , ... , £31 500

+d +d 8 more times

$31\,500 = 16\,200 + 9d$

$d = \dfrac{31500 - 16\,200}{9}$

$d = 1700$

now : $a_n = a + d(n-1)$

$a_2 = 16200 + (1700)(2-1)$

$a_2 = £17\,900$

(b) In model B, we can find r then use it to find salary in year 2:

Leave blank

Question 7 continued

year : 1 , 2 , ... , 10

salary : £16200 , a_2 , ... , £31 500

$\times r$ $\times r$ 8 more times

So : $31\,500 = 16200 \times r^{9}$ → multiplying by r 9 times.

$$r^9 = \frac{31\,500}{16\,200}$$

$$r = \sqrt[9]{\frac{31\,500}{16\,200}} = 1.07668\ldots$$

$$a_n = ar^{n-1}$$

$$a_2 = 16200(1.07668\ldots)^{2-1}$$

$$= 17\,442 = £17\,440 \text{ (nearest £10)}$$

(c) ① total for A :

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{10} = \frac{10}{2}(16\,200 + 31\,500)$$

$$= £238\,500$$

② total for B :

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{10} = \frac{16200(1-[1.07668\ldots]^{10})}{(1-[1.07668\ldots])}$$

$$= £231\,019$$

Leave blank

Question 7 continued

difference = 238 500 − 231 019

= 7481 = £7480 (nearest £10)

Leave blank

Question 7 continued

(Total 9 marks)

Q7

Leave blank

8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b\log_2 3$, where a and b are constants to be found.
(4)

(ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2\log_5(y + 1) - 1$$

(5)

Recall rules of logarithms ;

$\log_a(b) + \log_a(c) = \log_a(b \times c)$

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$\log a^b = b\log a$

(i) note !
Q wants final answer to include $\log_2 3$

take logs of both sides, (base 2)

$\log_2 8^{2x+1} = \log_2 6$

$\log_2 8^{2x+1} = \log_2 6$

$(2x+1)\log_2 8 = \log_2 6$

we know $8 = 2^3$

$2x+1(\log_2 8) = \log_2 6$

$(2x+1)(\log_2(2^3)) = \log_2 6$

$(2x+1)(3\log_2 2) = \log_2 6 \rightarrow \log_a a = 1$

$(2x+1)(3) = \log_2 6$

$\frac{(2x+1)(3)}{3} = \frac{\log_2 6}{3}$

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Question 8 continued

$$2x+1 = \frac{\log_2 6}{3}$$

$$2x+1 = \frac{\log_2 (2\times 3)}{3}$$

$$2x+1 = \frac{\log_2 (2\times 3)}{3}$$

$$2x+1 = \frac{\log_2 2 + \log_2 3}{3}$$

$$2x+1 = \frac{1 + \log_2 3}{3}$$

$$2x = \frac{1}{3} + \frac{\log_2 3}{3} - 1$$

$$2x = \frac{\log_2 3}{3} - \frac{2}{3}$$

divide both sides by 2 :

$$x = \frac{\log_2 3}{6} - \frac{1}{6}$$

$$x = \frac{\log_2 3}{6} - \frac{1}{3}$$

ALTERNATIVE SOLUTION :

$$8^{2x+1} = 6$$

we know $8 = 2^3$

$$(2^3)^{2x+1} = 6$$

use rule : $(a^b)^c = a^{b\times c}$

$$2^{3(2x+1)} = 6$$

$$2^{6x+3} = 6$$

Leave blank

Question 8 continued

take $\log_2$ of both sides :

$$\log_2 2^{6x+3} = \log_2 6$$

$$\log_2 2^{6x+3} = \log_2 6$$

$$(6x+3)\log_2 2 = \log_2 6$$

$$(6x+3)(1) = \log_2 6$$

$$6x+3 = \log_2(2\times 3)$$

$$6x+3 = \log_2(2\times 3)$$

$$6x+3 = \log_2 2 + \log_2 3$$

$$6x+3 = 1 + \log_2 3$$

make x subject :

$$6x+3 = 1 + \log_2 3$$

$$6x = 1 - 3 + \log_2 3$$

$$\frac{6x}{6} = \frac{-2+\log_2 3}{6}$$

$$x = \frac{-2}{6} + \frac{\log_2 3}{6}$$

$$x = -\frac{1}{3} + \frac{\log_2 3}{6}$$

(ii) $\log_5(7-2y) = 2\log_5(y+1) - 1$

$$\log_5(7-2y) - 2\log_5(y+1) = -1$$

$$\log_5(7-2y) - \log_5(y+1)^2 = -1$$

$$\log_5(7-2y) - \log_5(y+1)^2 = -1$$

Leave blank

Question 8 continued

$$\log_5\left(\frac{7-2y}{(y+1)^2}\right) = -1$$

use rule : $\log_a b = c \rightarrow a^c = b$

$$\log_5\left(\frac{7-2y}{(y+1)^2}\right) = -1$$

$$\frac{7-2y}{(y+1)^2} = 5^{-1}$$

use rule : $a^{-b} = \frac{1}{a^b}$

$$\frac{7-2y}{(y+1)^2} = \frac{1}{5}$$

multiply both sides by $(y+1)^2$

$$(y+1)^2\left(\frac{7-2y}{(y+1)^2}\right) = \left(\frac{1}{5}\right)(y+1)^2$$

$$7-2y = \frac{1}{5}(y+1)^2$$

expand $(y+1)^2$:

$$7-2y = \frac{1}{5}(y^2+2y+1)$$

$$7-2y = \frac{1}{5}(y^2+2y+1)$$

$$7-2y = \frac{1}{5}y^2 + \frac{2}{5}y + \frac{1}{5}$$

multiply both sides by 5 :

$$5 \times (7-2y) = \left(\frac{1}{5}y^2 + \frac{2}{5}y + \frac{1}{5}\right) \times 5$$

$$35 - 10y = y^2 + 2y + 1$$

Q8

(Total 9 marks)

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Question 8 continued

$y^2 + 2y + 1 + 10y - 35 = 0$

$y^2 + 12y - 34 = 0$ → factorise!

$1y^2 + 12y - 34 = 0$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-12 \pm \sqrt{12^2 - 4(1)(-34)}}{2(1)}$$

$$= 6 \pm \sqrt{70}$$

$y = 6 + \sqrt{70}$ $\quad$ $y = 6 - \sqrt{70}$ × reject:

$\log_2(6 - \sqrt{70} + 1)$ is -ve, log of a -ve value is not possible

Leave blank

9. (a) Show that the equation

$$\cos\theta - 1 = 4\sin\theta\tan\theta$$

can be written in the form

$$5\cos^2\theta - \cos\theta - 4 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < \frac{\pi}{2}$ → IN RADIANS !!

$$\cos 2x - 1 = 4\sin 2x \tan 2x$$

giving your answers, where appropriate, to 2 decimal places.

(4)

(a) $\cos\theta - 1 = 4\sin\theta\tan\theta$ → $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cos\theta - 1 = 4\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)$

$\cos\theta \times (\cos\theta - 1) = \frac{4\sin^2\theta}{\cos\theta} \times \cos\theta$

$\cos^2\theta - \cos\theta = 4\sin^2\theta$

Use identity : $\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta - \cos\theta = 4(1 - \cos^2\theta)$

$\cos^2\theta - \cos\theta = 4 - 4\cos^2\theta$

$5\cos^2\theta - \cos\theta - 4 = 0$

(b) $\cos 2x - 1 = 4\sin 2x \tan 2x$
$\cos\theta - 1 = 4\sin\theta\tan\theta$] same structure

SO: $5\cos^2 2x - \cos 2x - 4 = 0$ → FACTORISE !

$(5\cos 2x + 4)(\cos 2x - 1) = 0$

$5\cos 2x + 4 = 0$ or $\cos 2x - 1 = 0$

P 6 2 7 2 1 A 0 2 4 3 2

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Question 9 continued

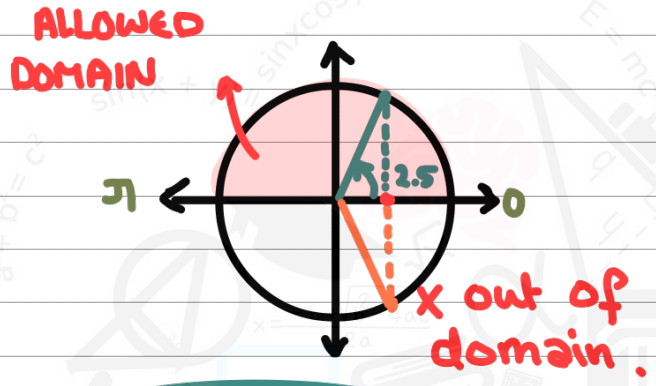
① $5\cos 2x + 4 = 0$, $2(0) \leq 2x < 2(\pi/2)$

$\cos 2x = \frac{-4}{5}$

ON CALCULATOR :

$2x = \cos^{-1}\left(\frac{-4}{5}\right) = 2.50$

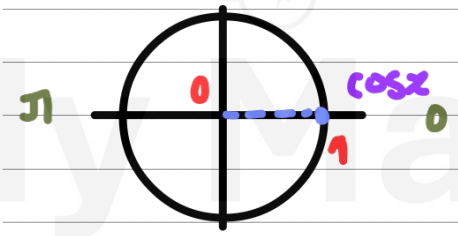
• note : there are no other angles on the unit circle within our given range where $\cos 2x = -4/5$



so $2x = 2.50$

② $\cos 2x - 1 = 0$

$\cos 2x = 1$



so : $2x = 0$ when $\cos 2x = 1$

$2x = 0, 2.50$

$x = 0, 1.25$

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Question 9 continued

Q9

(Total 8 marks)

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10.

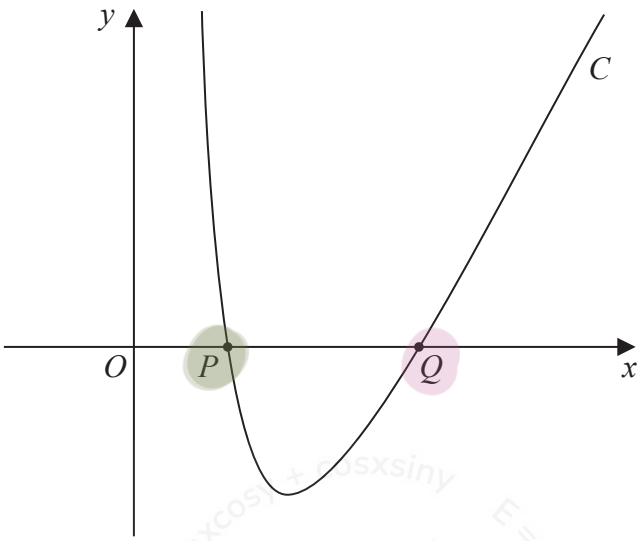

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{36}{x^2} + 2x - 13 \qquad x > 0$$

Using calculus,

(a) find the range of values of x for which $\mathrm{f}(x)$ is increasing,

(4)

(b) show that $\int_2^9 \left(\frac{36}{x^2} + 2x - 13\right) \mathrm{d}x = 0$

(4)

The point $P(2, 0)$ and the point $Q(6, 0)$ lie on C.

Given $\int_2^6 \left(\frac{36}{x^2} + 2x - 13\right) \mathrm{d}x = -8$

(c) (i) state the value of $\int_6^9 \left(\frac{36}{x^2} + 2x - 13\right) \mathrm{d}x$

(ii) find the value of the constant k such that $\int_2^6 \left(\frac{36}{x^2} + 2x + k\right) \mathrm{d}x = 0$

(3)

(a) Where f(x) is increasing, there is a positive gradient : $f'(x) > 0$

• Find $f'(x)$ of $f(x)$:

$$f(x) = \frac{36}{x^2} + 2x - 13$$

Leave blank

Question 10 continued

$f(x) = 36x^{-2} + 2x - 13$

$f(x) = ax^n \rightarrow f'(x) = nax^{n-1}$ differentiation rule :)

$f'(x) = (-2)36x^{-2-1} + (1)2x^{1-1} - (0)13x^{0-1}$

$f'(x) = -72x^{-3} + 2$

$= \frac{-72}{x^3} + 2$

$f'(x) > 0$

$\left[\frac{-72}{x^3}\right] + 2 > 0$

$2 > \frac{72}{x^3}$

• multiply both sides by x^3 :

$x^3 \times (2) > \left(\frac{72}{x^3}\right) \times x^3$

$2x^3 > 72$

• divide by 2 :

$\frac{2x^3}{2} > \frac{72}{2}$

$x^3 > 36 \rightarrow x > \sqrt[3]{36}$

interval where $f(x)$ is increasing!

(b) $\int_2^9 \frac{36}{x^2} + 2x - 13 \, dx$

$= \int_2^9 36x^{-2} + 2x - 13 \, dx$

REMEMBER!

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1}}$$

Leave blank

Question 10 continued

$$\left[\frac{36x^{-2+1}}{-2+1} + \frac{2x^{1+1}}{1+1} - \frac{13x^{0+1}}{0+1}\right]_2^9$$

$$\left[\frac{36x^{-2+1}}{-2+1} + \frac{2x^{1+1}}{1+1} - \frac{13x^{0+1}}{0+1}\right]_2^9$$

$$\left[-36x^{-1} + x^2 - 13x\right]_2^9 \quad \left(\text{RULE: } x^{-a} = 1/x^a\right)$$

$$\left[\frac{-36}{x} + x^2 - 13x\right]_2^9$$

$$= \left[\frac{-36}{9} + 9^2 - 13(9)\right] - \left[\frac{-36}{2} + 2^2 - 13(2)\right]$$

$$= (-4 + 81 - 117) - (-18 + 4 - 26)$$

$$= -40 - (-40) = 0$$

(c) (i) If :

$$\boxed{\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx}$$

So :

$$\int_2^9 \frac{36}{x^2} + 2x - 13\,dx = \int_2^6 \frac{36}{x^2} + 2x - 13\,dx + \int_6^9 \frac{36}{x^2} + 2x - 13\,dx$$

$$0 = -8 + \int_6^9 \frac{36}{x^2} + 2x - 13\,dx$$

$$\int_6^9 \frac{36}{x^2} + 2x - 13\,dx = 8$$

P 6 2 7 2 1 A 0 3 0 3 2

Leave blank

Question 10 continued

(ii) $\int_2^6 \frac{36}{x^2} + 2x + k \, dx$

INTEGRATE : (see steps in (b))

$\left[-\frac{36}{x} + x^2 + kx \right]_2^6$

$= \left[-\frac{36}{6} + 6^2 + k(6) \right] - \left[-\frac{36}{2} + 2^2 + k(2) \right]$

$= -6 + 36 + 6k - (-18 + 4 + 2k)$

$= 30 + 6k + 14 - 2k$

$= 44 + 4k$

$\int_2^6 \frac{36}{x^2} + 2x + k \, dx = 0$

$44 + 4k = 0$

$4k = -44$

$k = -11$

Leave blank

Question 10 continued

Q10

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END